Gauge-Invariant Atmospheric Energy Flux: Extracting Key Structures in Atmospheric Energy Transport

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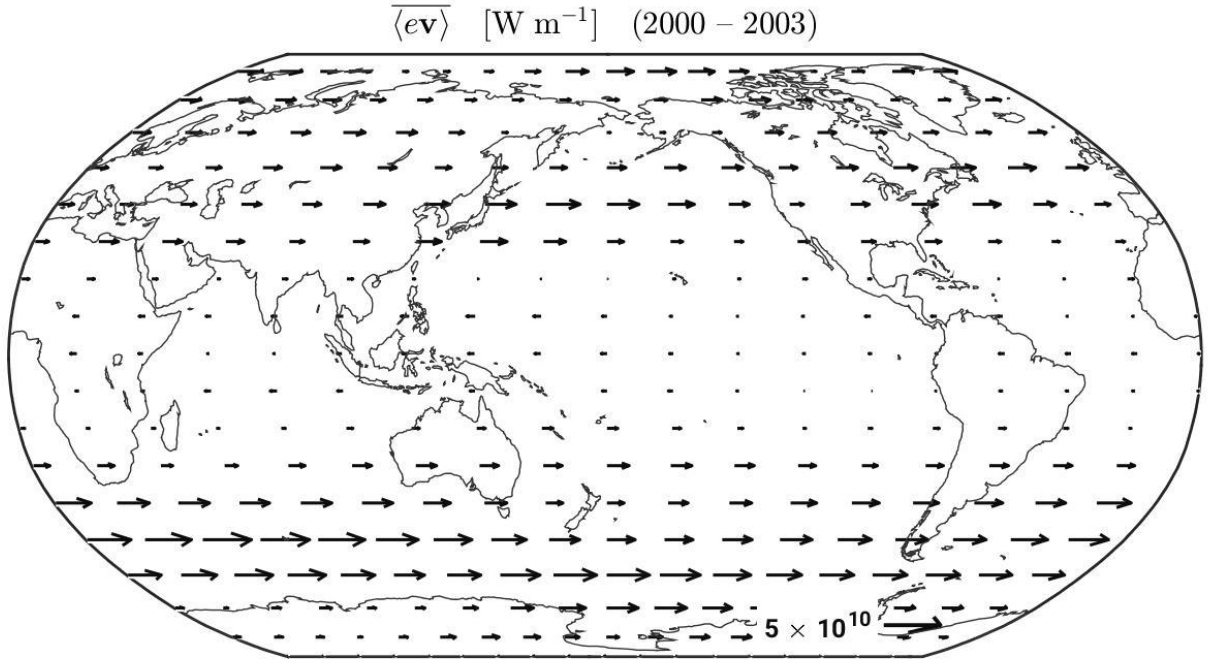
Where energy comes from, where it goes, and how it is transported

- Workshop theme: understand Earth's energy imbalance.
- Key diagnostic: vertically integrated atmospheric energy flux.

$$\langle e\mathbf{v} \rangle \equiv \int_0^{p_s} e\mathbf{v} \frac{dp}{g} \qquad e \equiv c_p T + gz + L_v q_v - L_i q_i + \frac{1}{2}(u^2 + v^2)$$

- This flux looks simple but is subtle to interpret.
- **Main Goal**: separate meaningful transport from arbitrary structure.

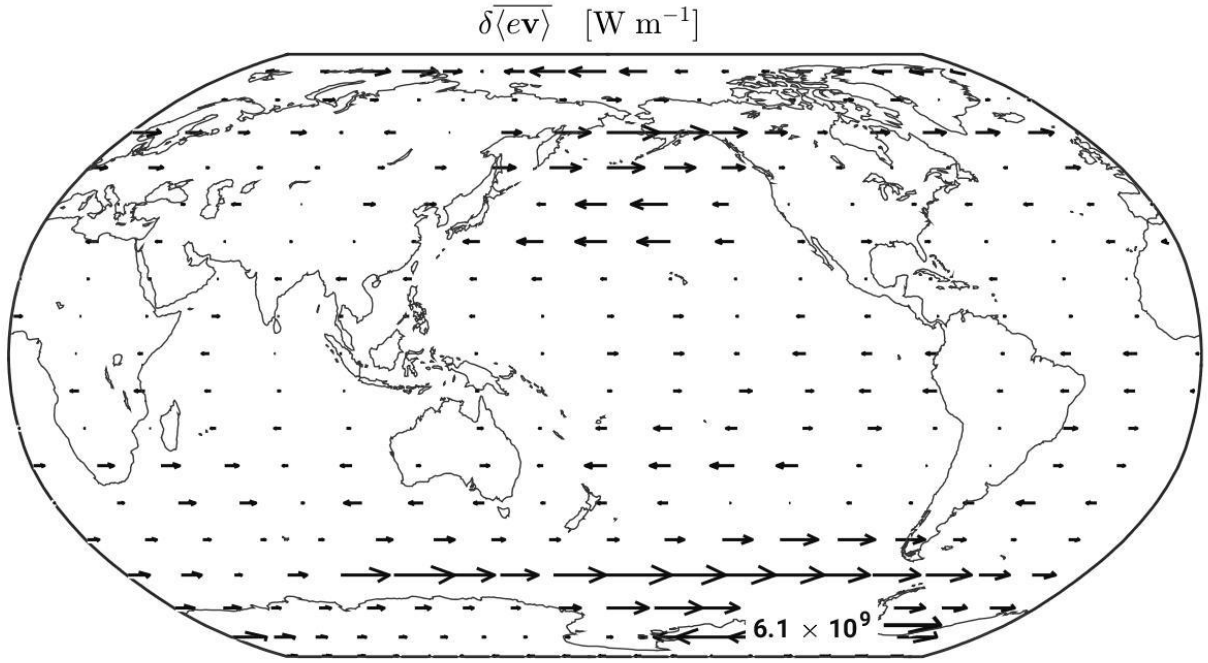
The total flux mostly shows mass flow, not energy-transport structure



With MERRA2

- Most arrows point zonally.
- Midlatitude westerlies produce eastward flux.
- Tropical weak easterlies produce weak westward flux.
- This quantity is not informative.

Raw temporal changes remain dominated by unhelpful zonal structure



$$\delta\left(e\mathbf{v}\right) = \left(e\mathbf{v}\right)_{2020--2023} - \left\langle e\mathbf{v}\right\rangle_{2000--2003}$$

- The difference field remains mostly zonal.
- The apparent signal is visually hard to interpret.
- The key problem is not just noise; much of the signal is not physical.

Helmholtz decomposition isolates the budget-relevant flux

- Helmholtz decomposition: split into **divergent** and **rotational** parts (e.g., Trenberth & Stepaniak, 2003; Lintner and Boos, 2019)

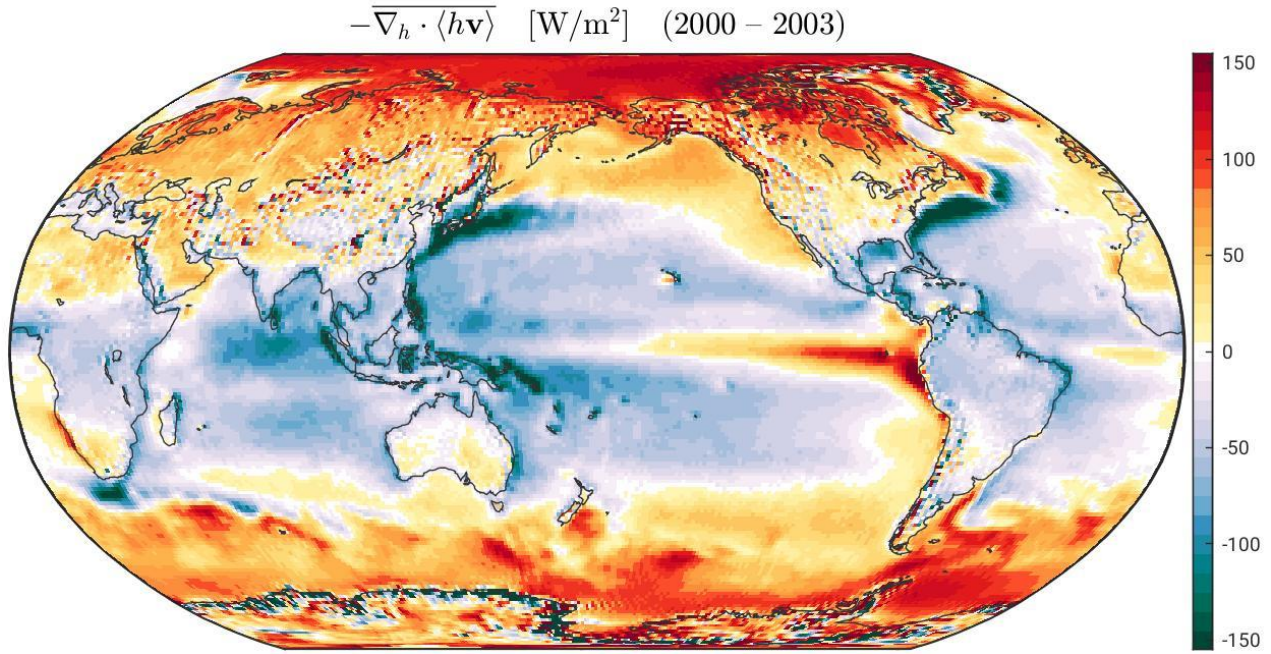
$$\mathbf{F} = \mathbf{F}_D + \mathbf{F}_R \qquad \mathbf{F} \equiv \langle \epsilon \mathbf{v} \rangle$$

$$\nabla_h \chi = \mathbf{F}_D$$

$$\nabla_h^2 \chi = \nabla_h \cdot \mathbf{F} = \nabla_h \cdot \langle \epsilon \mathbf{v} \rangle \simeq S - OLR - O$$

$$\chi = \mathcal{L}^{-1}(S - OLR - O)$$

The budget-relevant divergent flux is tiny compared with the total flux



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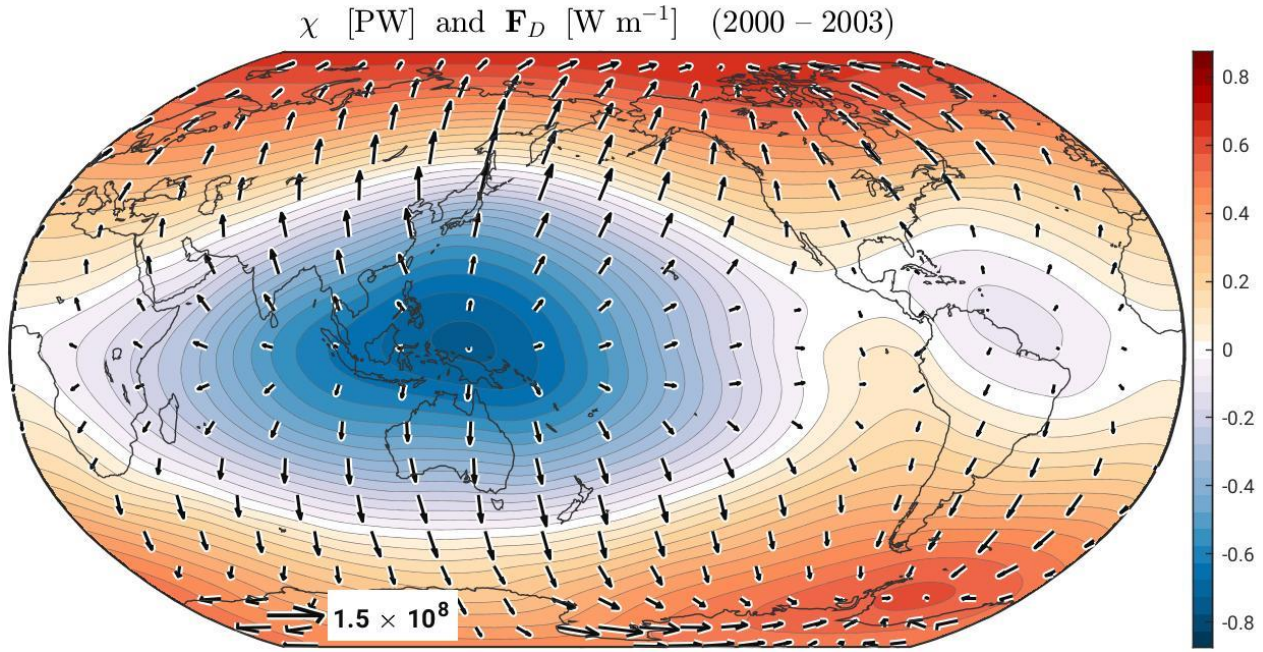

- The divergent flux magnitude is order 10^8.

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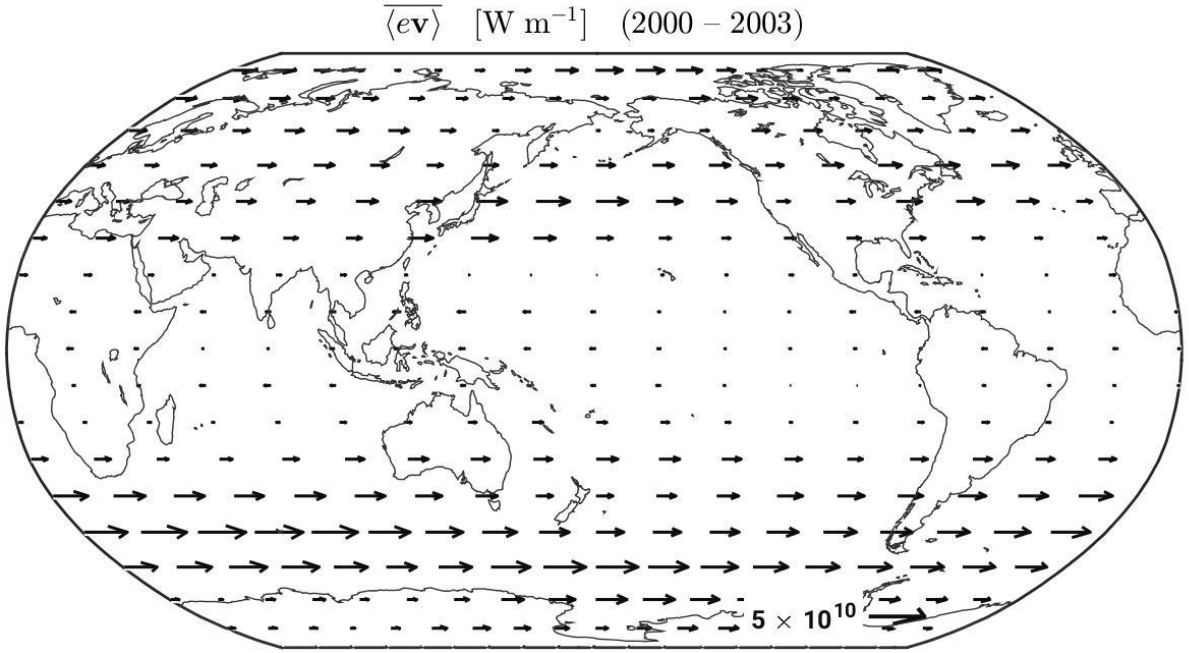

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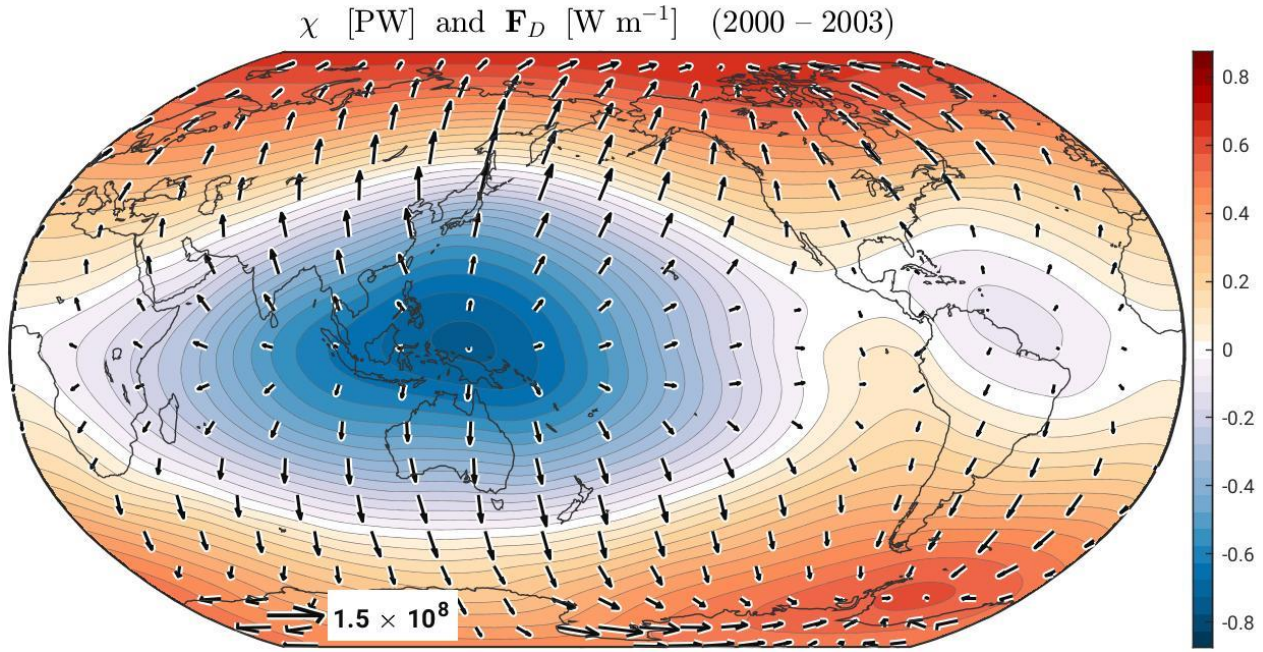
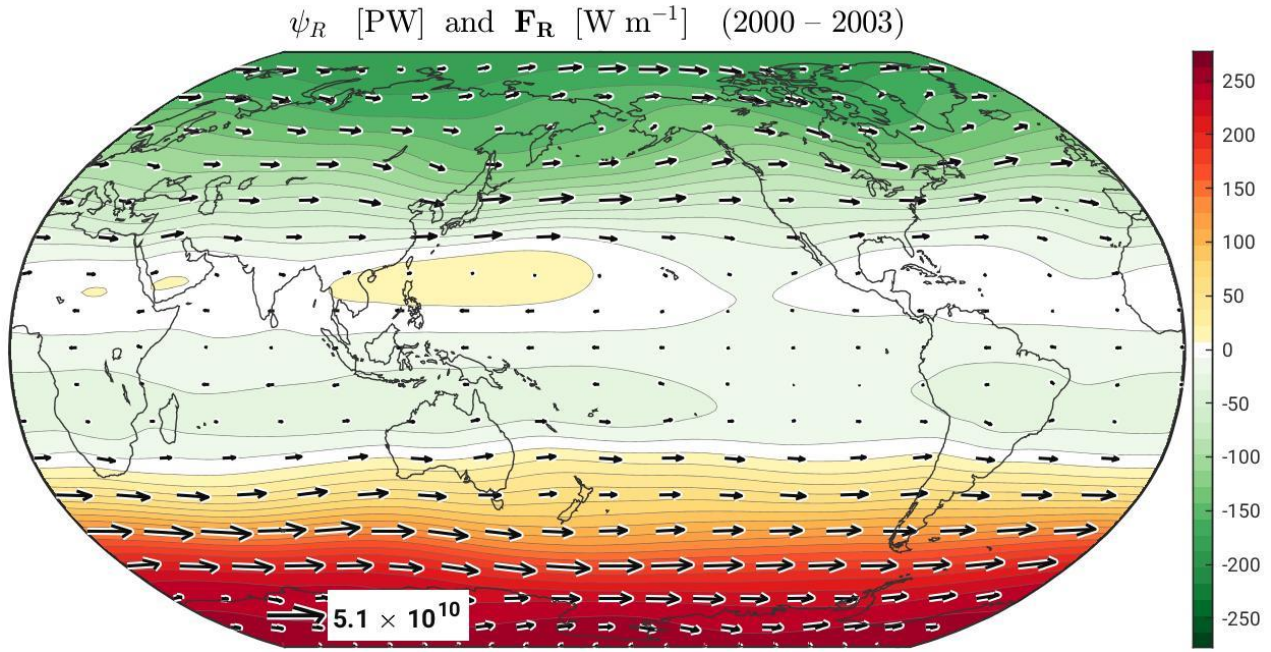


- The divergent flux magnitude is order 10^8.
- The total flux magnitude is much larger.
- The divergent component is more physically tied to sources and sinks.
- How about the rotational component?

The rotational flux is large but still looks physically opaque

$$\mathbf{F}_R = \mathbf{F} - \mathbf{F}_D$$



- The rotational flux is computed as the residual.
- The flux is tangent to streamfunction contours.
- Its magnitude is order 10^{10}.

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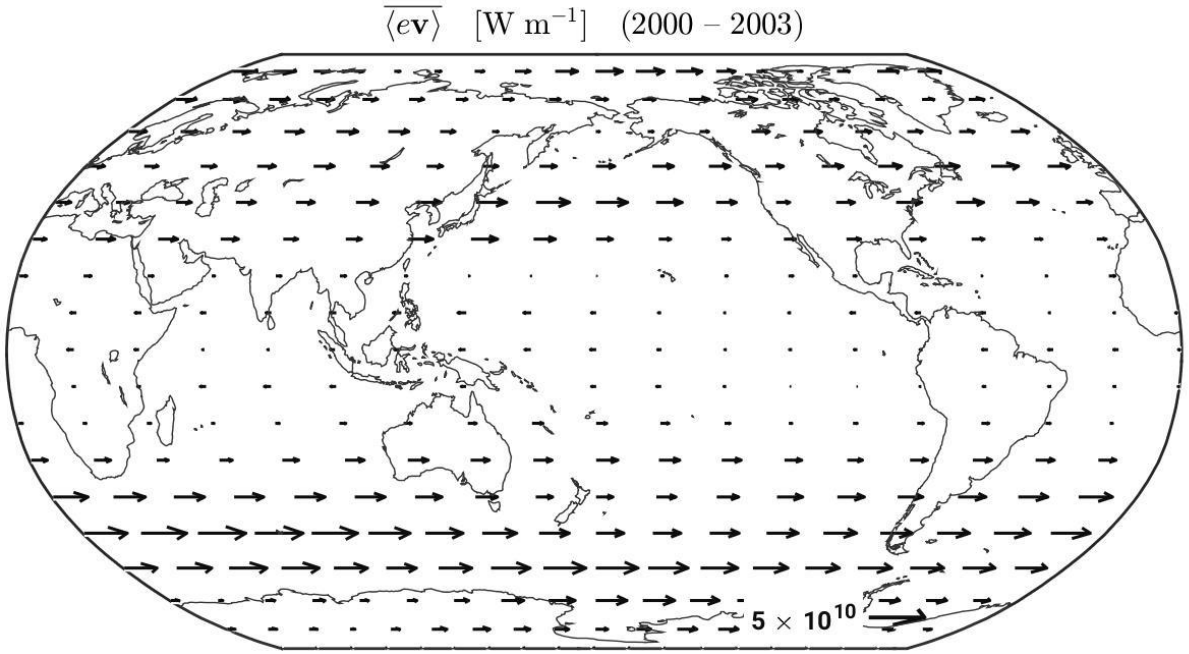

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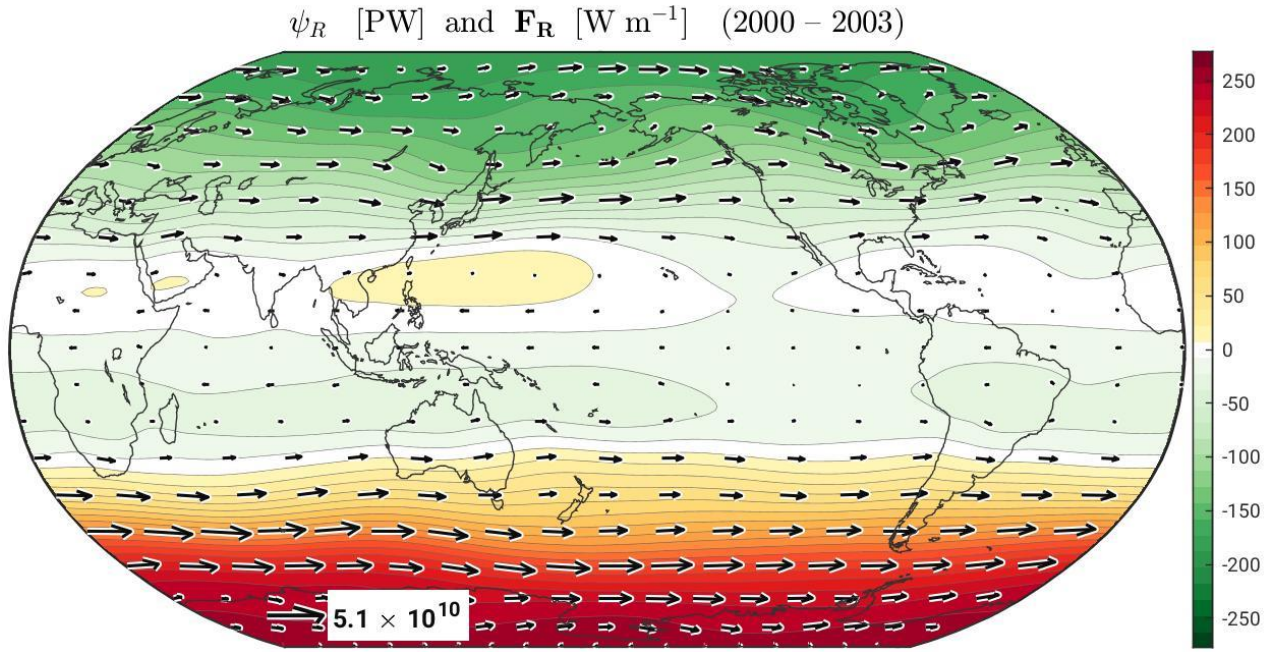


- The rotational flux is computed as the residual.
- The flux is tangent to streamfunction contours.
- Its magnitude is order 10^{10}.
- Much of this large signal is **artificial** or **arbitrary**.

Energy flux depends on the arbitrary energy reference level

- Energy is defined **relative to a reference state**.
  - Geopotential energy depends on a reference height.
  - Enthalpy can include an additive constant.

$$m = (1 - q_g - q_l - q_s)c_a(T_a - T_{00}) + (q_l + q_g + q_s)c_l(T_s - T_{00}) + L_v(T_a)q_g - L_f(T_s)q_s + (q_l + q_g + q_s)h_0 + \phi.$$

$$e \equiv c_p T + gz + L_v q_v - L_i q_i + \frac{1}{2}(u^2 + v^2)$$

Mayer et al. (2017)

- This difference doesn't matter for the energy conservation.
- **Energy changes** are observable, not absolute values.
- Flux depends on the absolute energy convention.

Gauge-Variant vs. Gauge Invariant Quantities

- **Gauge variance**: dependence on an arbitrary reference level choice.
- **Gauge-invariant** quantities do not depend on that choice.
- Physically meaningful conclusions should be based on gauge-invariant quantities.
- We need to separate the gauge-invariant components from the gauge-variant component in the energy flux.

Gauge Transformation

- **Guage Transformation**: an operation that adds a gauge element to an energy-related quantity while preserving the energy conservation law.

- Analogous to a change of coordinates.

$$e \longmapsto e^{*} \equiv e + c$$

Gauge Transformation: Changing a Reference Constant

$$e \longmapsto e^* \equiv e + c$$

$$(e^*\mathbf{v}) = (e\mathbf{v}) + c\,(\mathbf{v})$$

Gauge-Variant

$$\nabla_h \cdot (\mathbf{v}) = 0$$

- The gauge-variant component is completely rotational.
- The gauge-variant component is projected onto the vertically integrated mass flux field.

Projection estimates the gauge-variant rotational flux

$$(\epsilon^* \mathbf{v}) = \mathbf{F}_D^{gi} + \mathbf{F}_R^{gi} + \mathbf{F}_R^{gv}$$

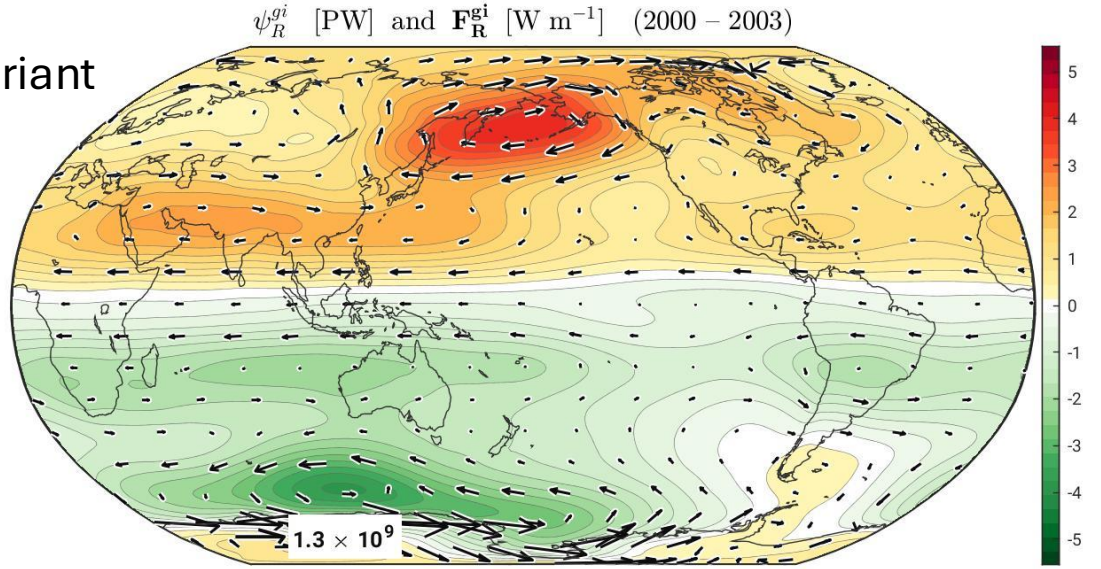
$$\mathbf{F}_R^{gv} = c\mathbf{M}, \qquad \mathbf{M} \equiv (\mathbf{v})$$

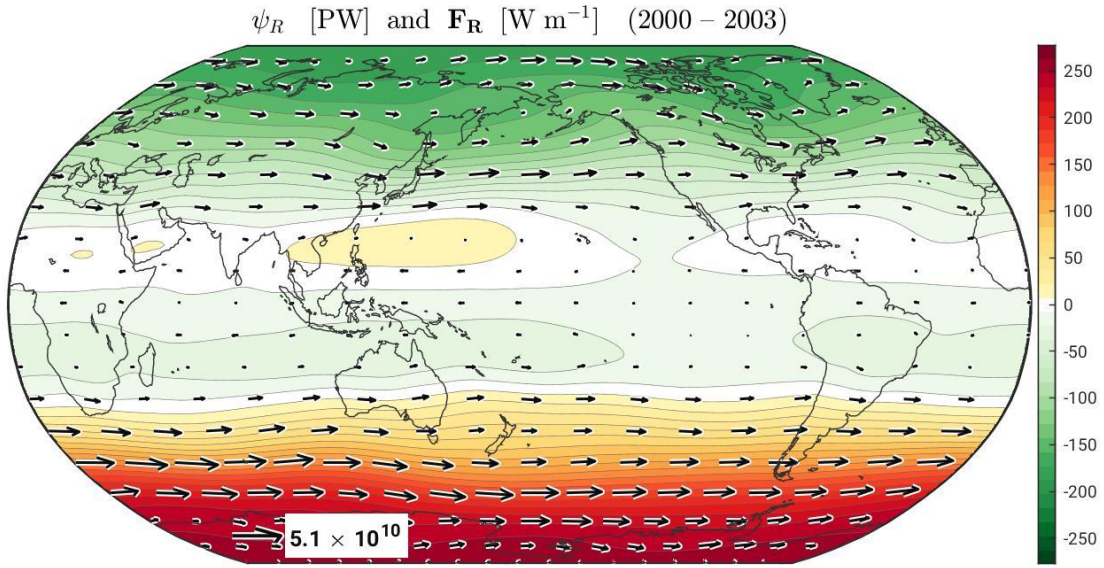
$$[\mathbf{A}, \mathbf{B}] = \int_\Omega \mathbf{A} \cdot \mathbf{B}\, dA$$

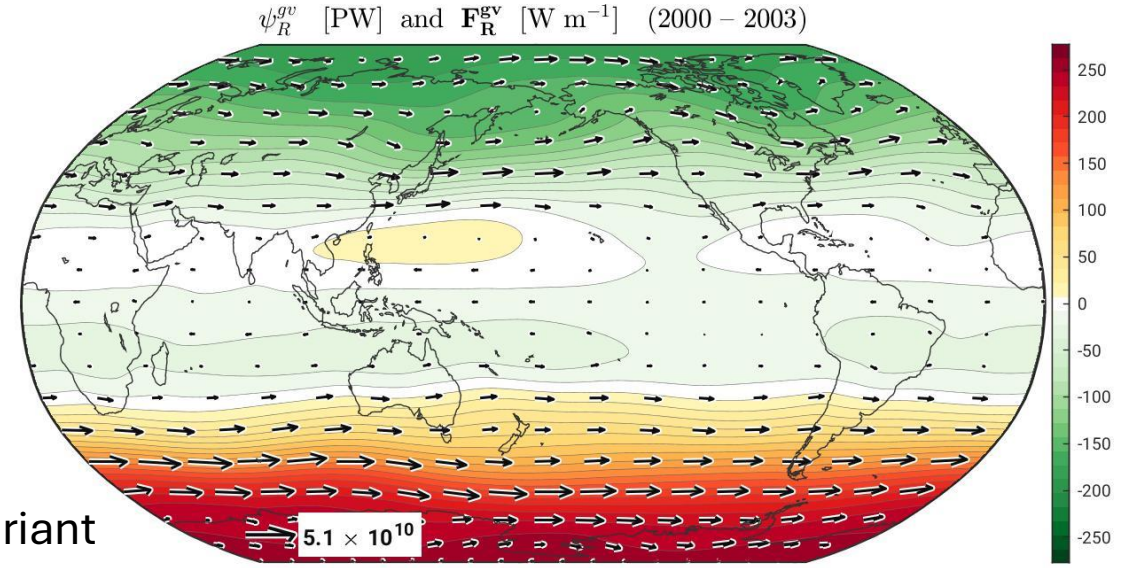
$$c = \frac{[\mathbf{F}_R, \mathbf{M}]}{[\mathbf{M}, \mathbf{M}]}$$

Most apparent rotational flux is gauge variant

Gauge-invariant



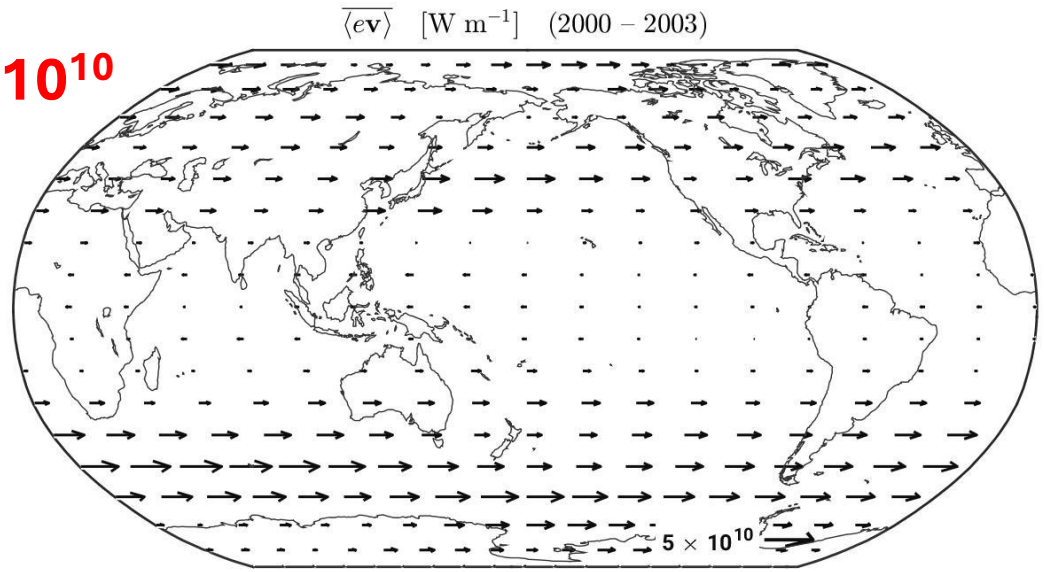




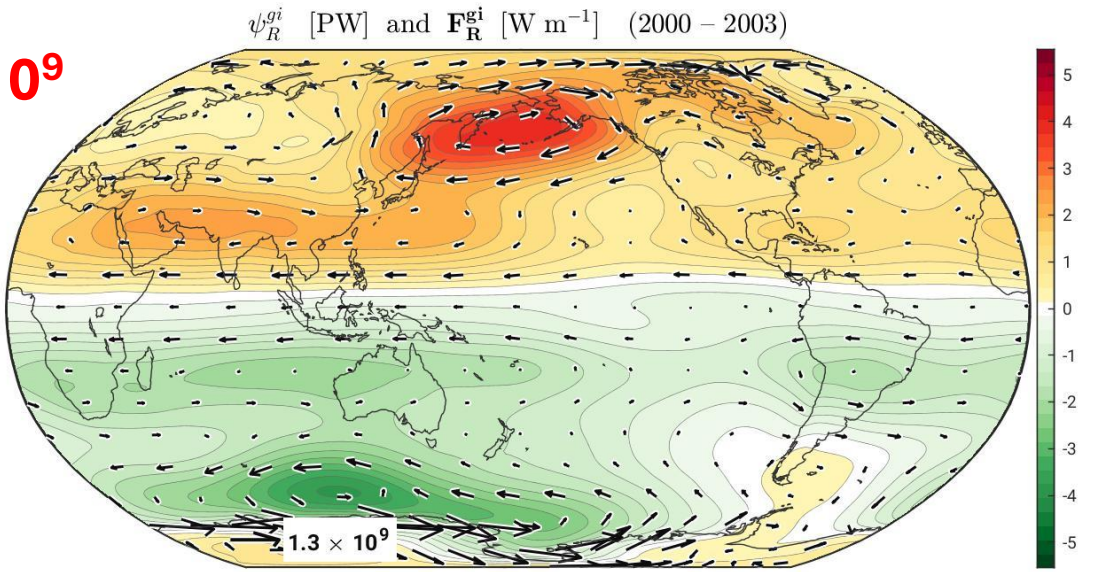
Gauge-variant

Total Flux = Divergent + GI Rotational + GV Rotational

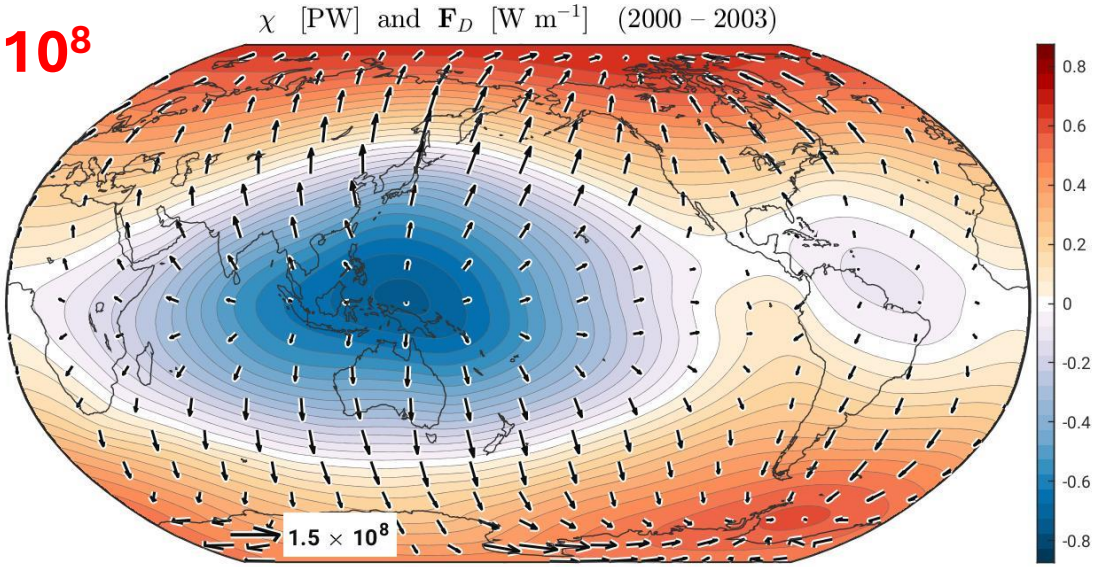
~10^{10}



~10^{9}



~10^{8}



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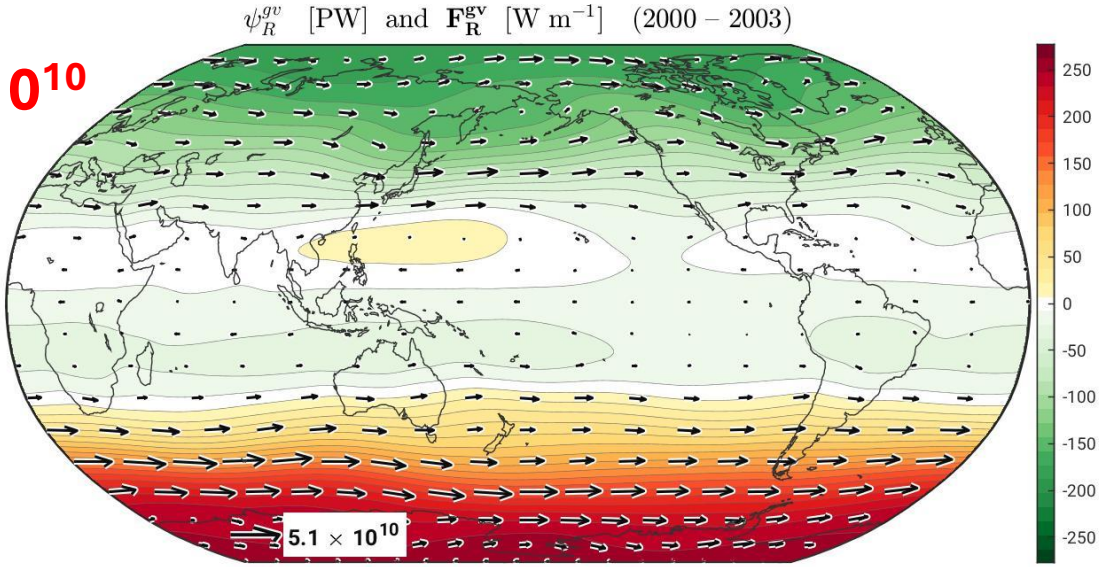


A reference-energy shift removes the gauge-variant flux

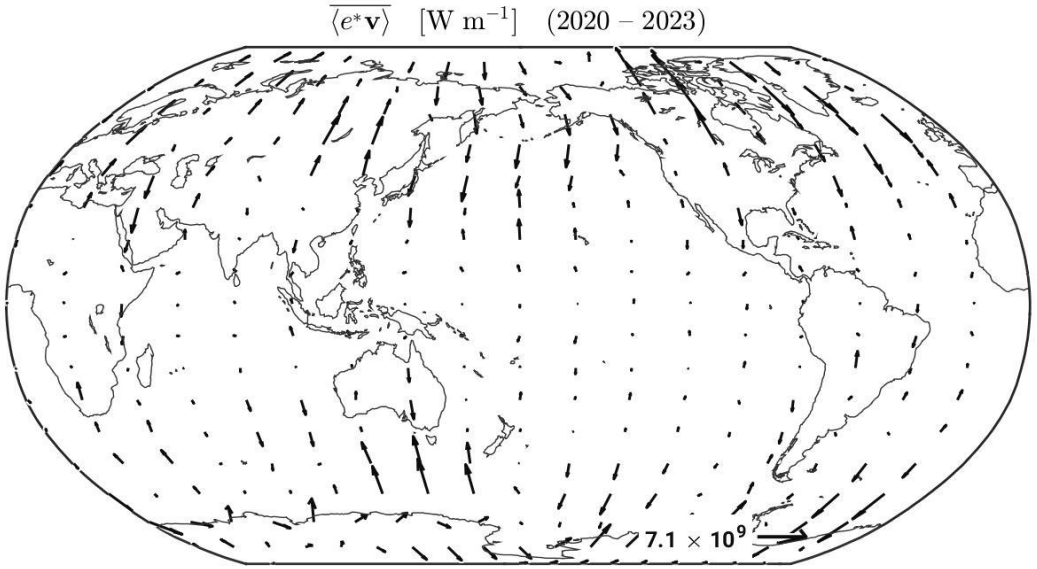
$$e \equiv c_p T + gz + L_v q_v - L_i q_i + \frac{1}{2}(u^2 + v^2)$$

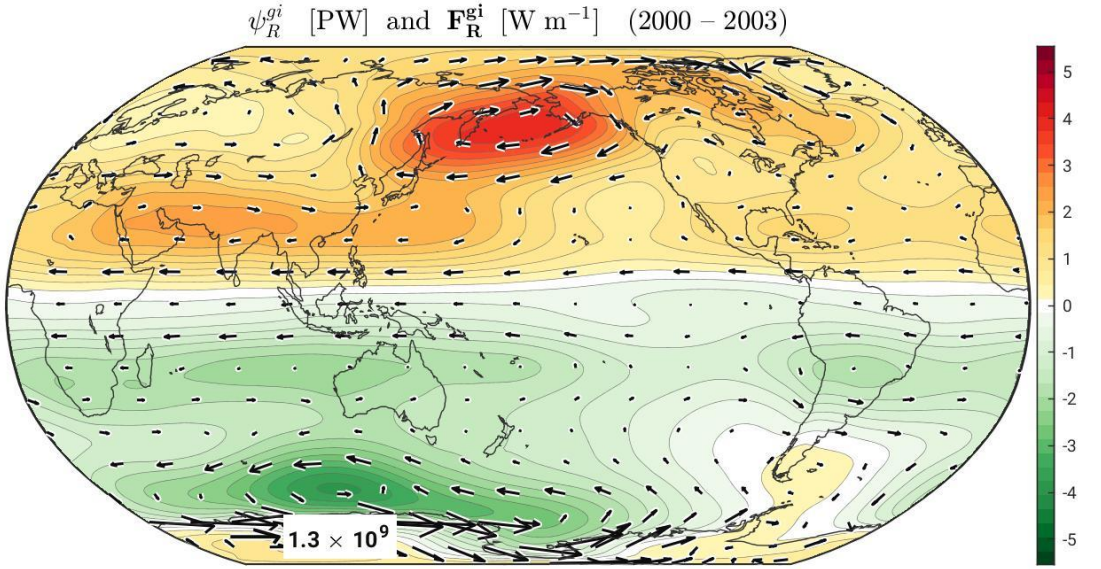
$$\langle e\mathbf{v} \rangle = \langle e\mathbf{v} \rangle_D + \langle e\mathbf{v} \rangle_R^{gi} + c \langle \mathbf{v} \rangle$$

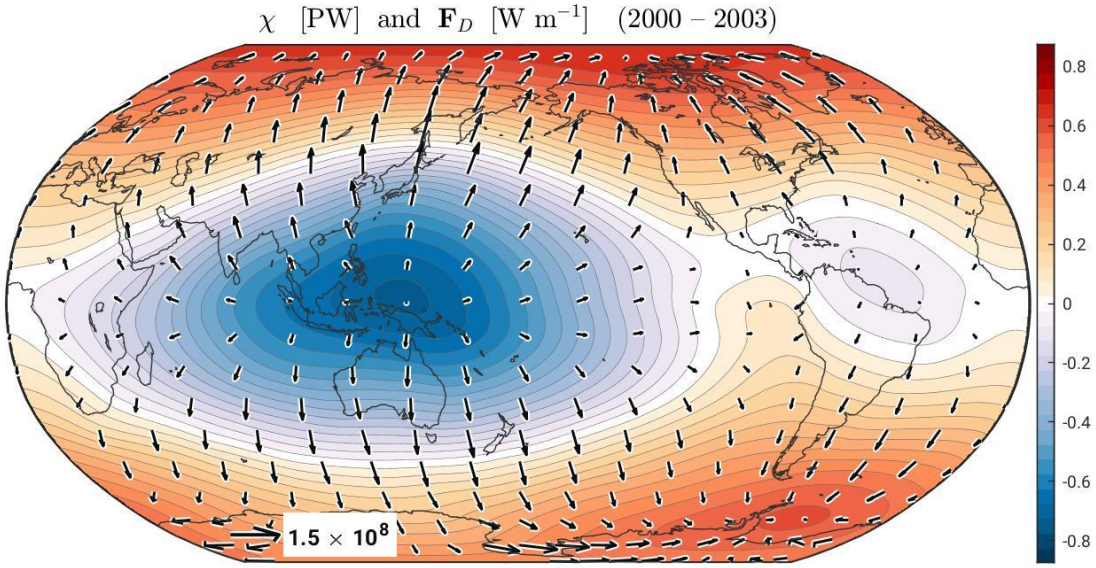
$$e^* \equiv c_p T + gz + L_v q_v - L_i q_i + \frac{1}{2}(u^2 + v^2) - c$$

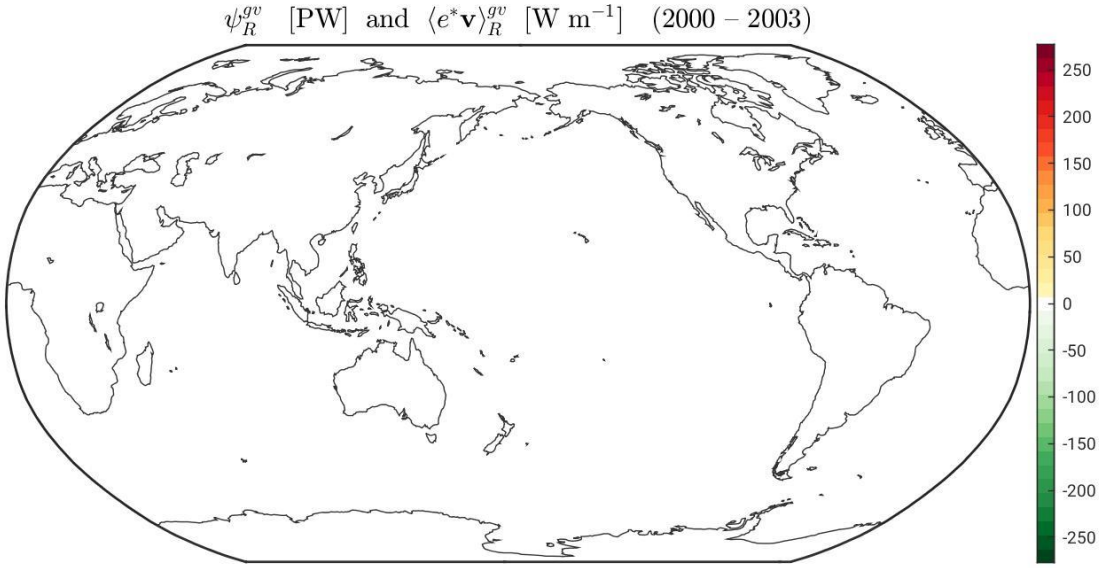
$$\begin{aligned} \langle e^* \mathbf{v} \rangle &= \langle e\mathbf{v} \rangle - c \langle \mathbf{v} \rangle \\ &= \langle e\mathbf{v} \rangle_D + \langle e\mathbf{v} \rangle_R^{gi} \end{aligned}$$

A reference-energy shift removes the gauge-variant flux

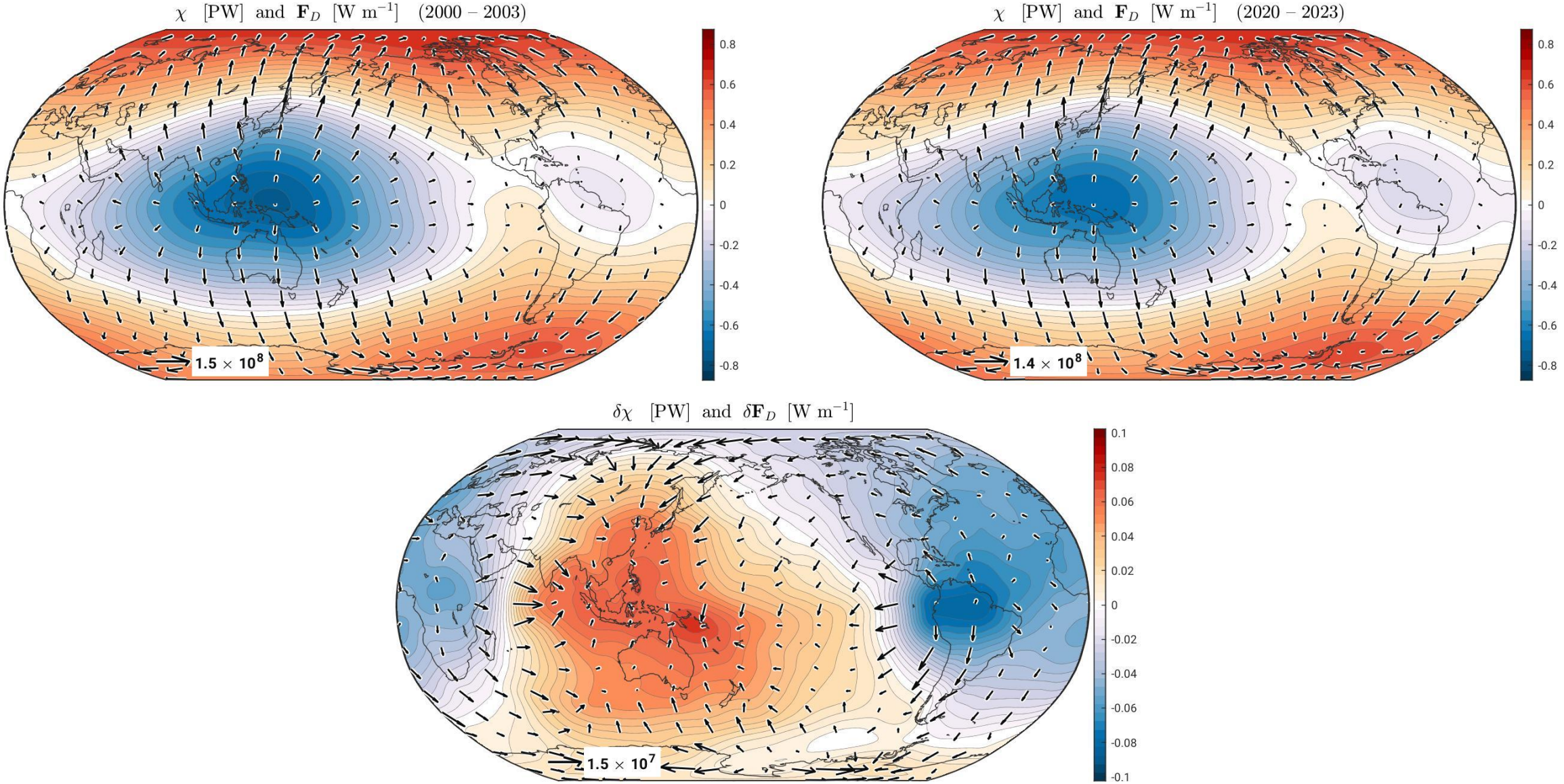




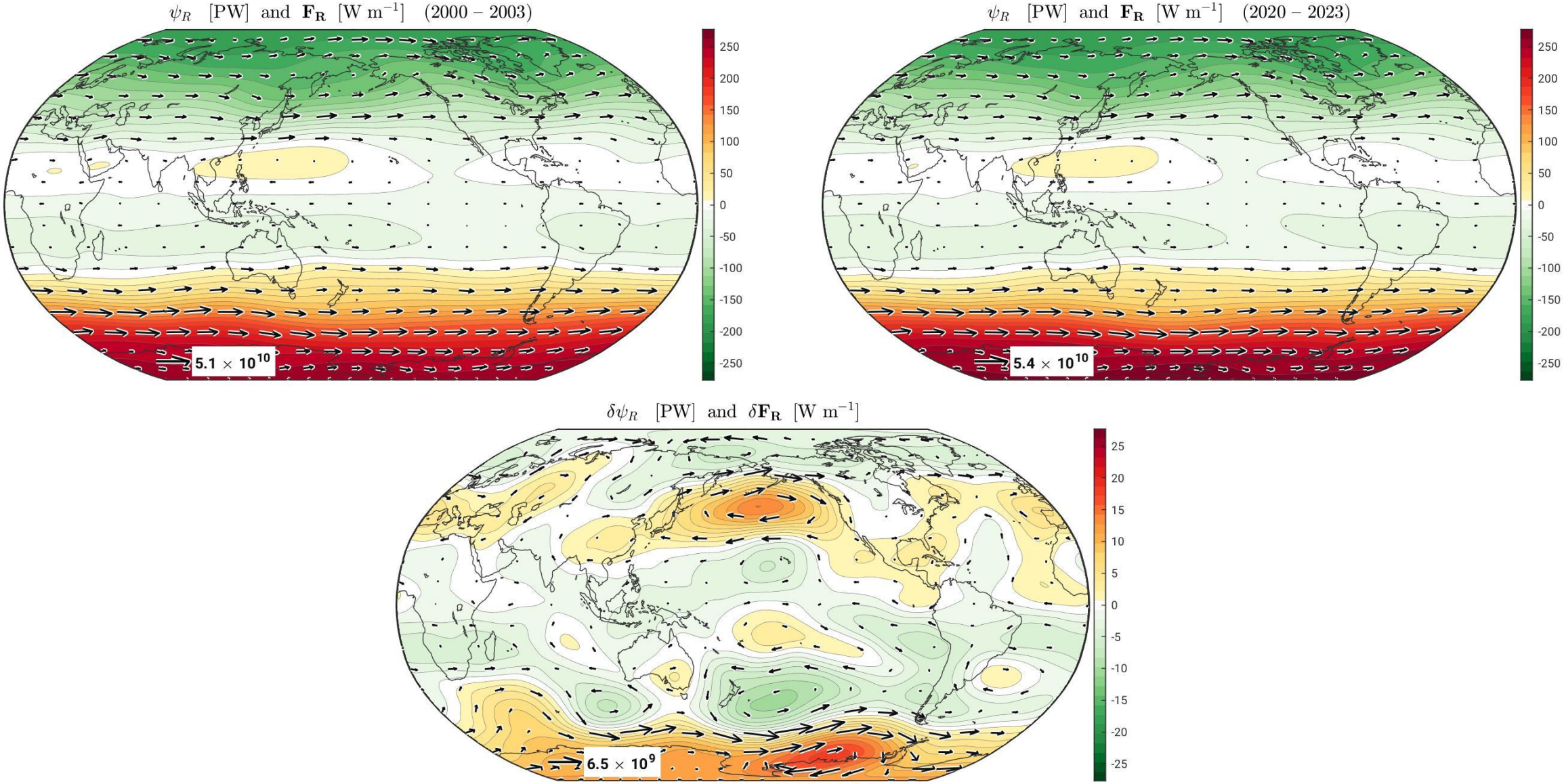




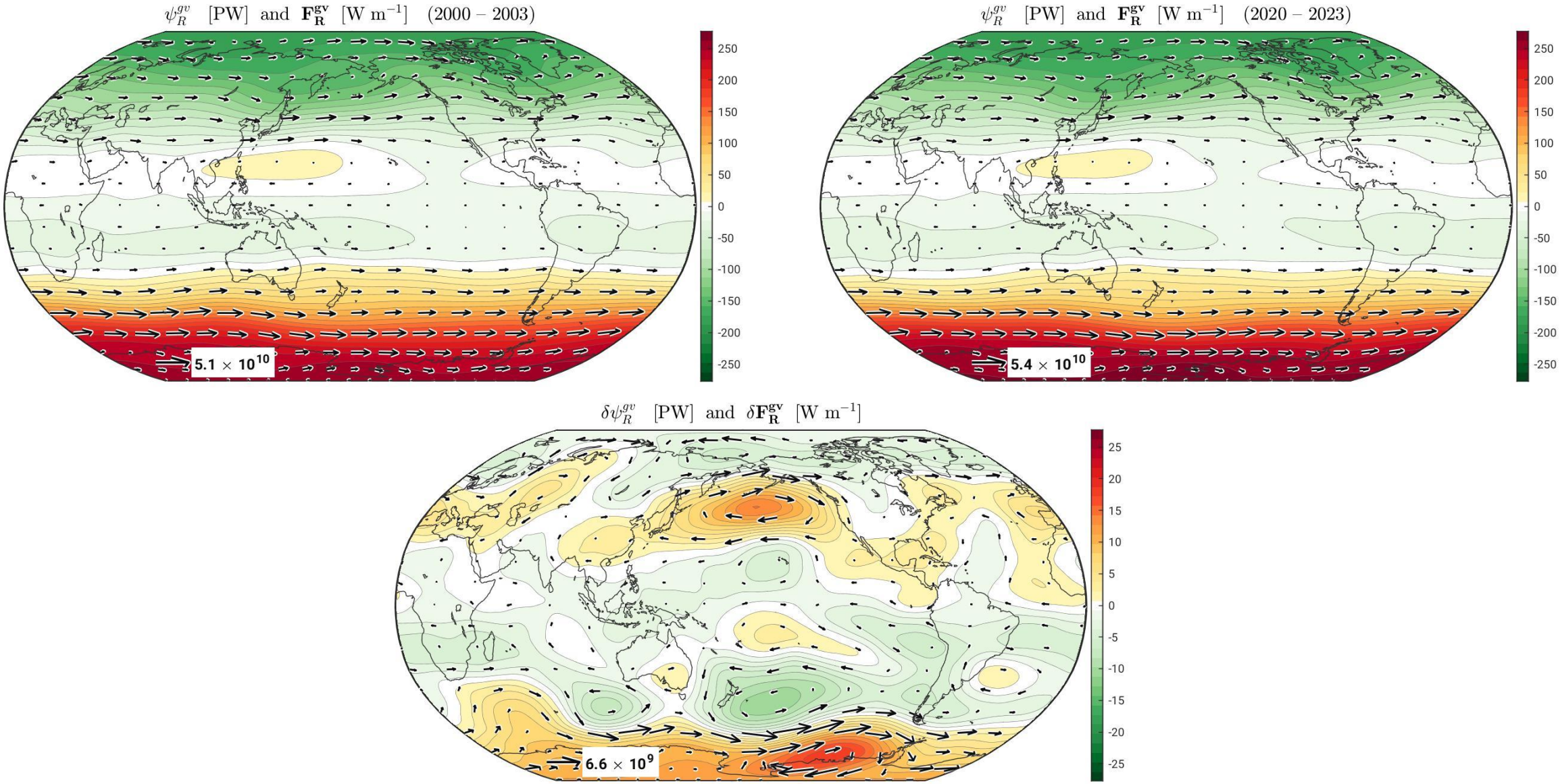
Gauge-invariant changes give a cleaner view of temporal variability



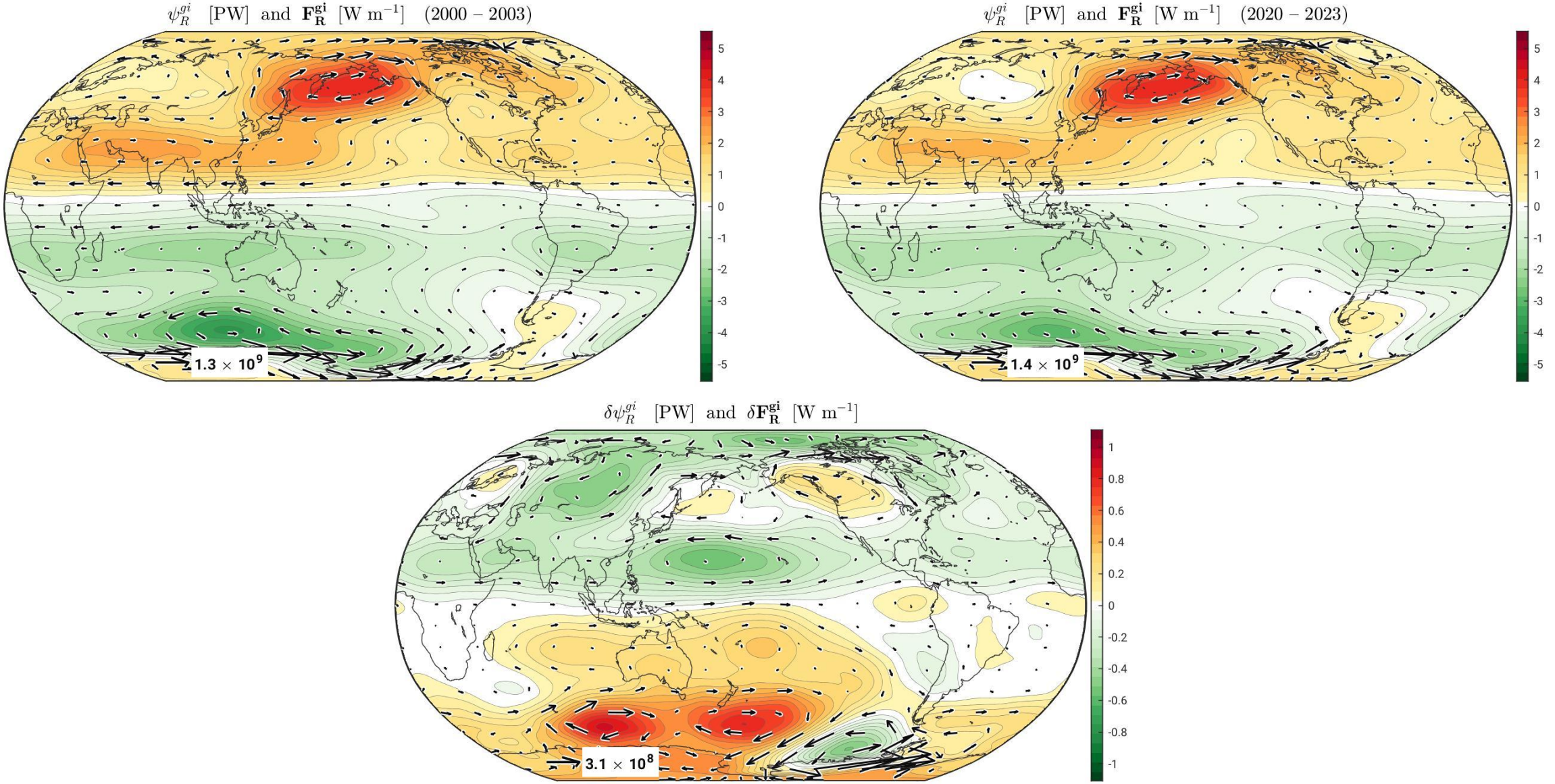
Most of the raw rotational component signals are arbitrary



Gauge-variant components dominate the rotational signals



The remaining rotational flux reveals hidden circulation pathways



Summary

- Total energy flux is dominated by arbitrary gauge-dependent structure.
- Divergent flux, that is gauge-invariant, connects directly to the global energy budget.
- Gauge-invariant rotational flux reveals additional circulation pathways.
- Temporal differences should be analyzed after removing gauge-variant flux.
- Removing gauge-dependent structure makes atmospheric energy-flux analysis more physically meaningful