Analyzing the numerical stability of atmosphere-surface coupling methods for momentum fluxes

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Summary

This work investigates numerical instability of explicit coupling schemes for atmospheric surface stresses, using a simple model including dynamic and Coriolis effects. This model can be analytically solved to estimate the maximum stable time step, and it can be used as a numerical test bed for more stable, alternative surface coupling schemes.

Boundary Layer Equations

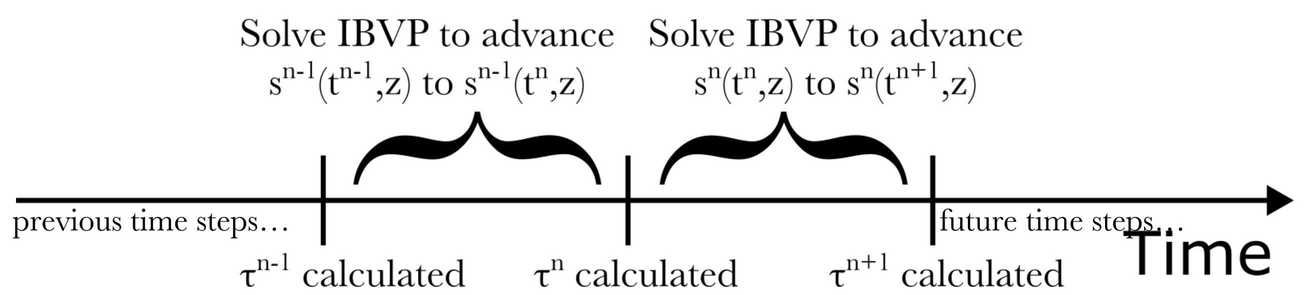
Consider the following initial boundary value problem (IBVP) over time t and altitude z at each coupling time step of size Δt:

$$\frac{\partial s^n(t,z)}{\partial t} = K\frac{\partial^2 s^n(t,z)}{\partial z^2} - \left[\frac{1}{\eta} + i\,f\right]\left(s^n(t,z) - u_g\right)$$

$$-K\frac{\partial s^n(t,z)}{\partial z} = \frac{\tau^n}{\rho} \quad @\ z = z_{bot}$$

$$\lim_{z\to\infty} s^n(t,z) = u_g \qquad s^n(t^n,z) = s^{n-1}(t^n,z)$$

Here $s^n(t,z) = u^n(t,z) + i\,v^n(t,z)$ approximates the horizontal winds between time $t^n = n\Delta t$ and $t^{n+1} = (n+1)\Delta t$. The stress τ^n is calculated once per time step, using the wind at height z_{bot} from the previous time step:



This parameterization of τ^n is used for numerical explorations:

$$\tau^n = -\rho\, C_d \sqrt{|s^{n-1}(t^n,z_{bot})|^2 + u_{gust}^2} \times s^{n-1}(t^n,z_{bot})$$

Stability Conditions

If the Coriolis parameter $f \approx 0$, e.g. near the equator, the condition for stability can be approximated using

$$\Delta t < \eta\left[\mathrm{erf}^{-1}\left(\frac{\sqrt{K}\rho}{\sqrt{\eta}\left(\frac{d|\tau|}{d|s|}\right)}\right)\right]^2$$

If the argument to the inverse error function is greater than 1, the maximum stable time step is unlimited. A simpler, sufficient condition for stability is given by

$$\Delta t < \frac{4\,K\,\rho^2}{\pi\left(\frac{d|\tau|}{d|s|}\right)^2} = \frac{K}{\pi\left(u_*\frac{du_*}{du_{bot}}\right)^2}$$

where u_* is the friction velocity and $u_{bot} = |s|$ is the lowest resolved wind speed in the atmosphere model. More general conditions exist for $f \neq 0$, but in terms of infinite series.

Numerical Instability

For parameter values meant to represent a convectively stable boundary layer over a rough surface, only a short atmosphere-land coupling time step ($< \sim 10$ min) avoids producing spurious oscillations related to instability (see Fig. 1). In this case the oscillations have a large effect on wind speeds in the bottom 150m of the 2km vertical domain, and would have a large effect on surface energy and mass fluxes calculated by Monin-Obukhov similarity theory.

We can test a variety of alternative schemes to remove these oscillations (see Fig. 2), including using an implicit predictor-corrector scheme to calculate stresses, or updating stress in the atmosphere model time step. The implicit method conserves momentum exactly, but it requires 1 time step to "spin up".

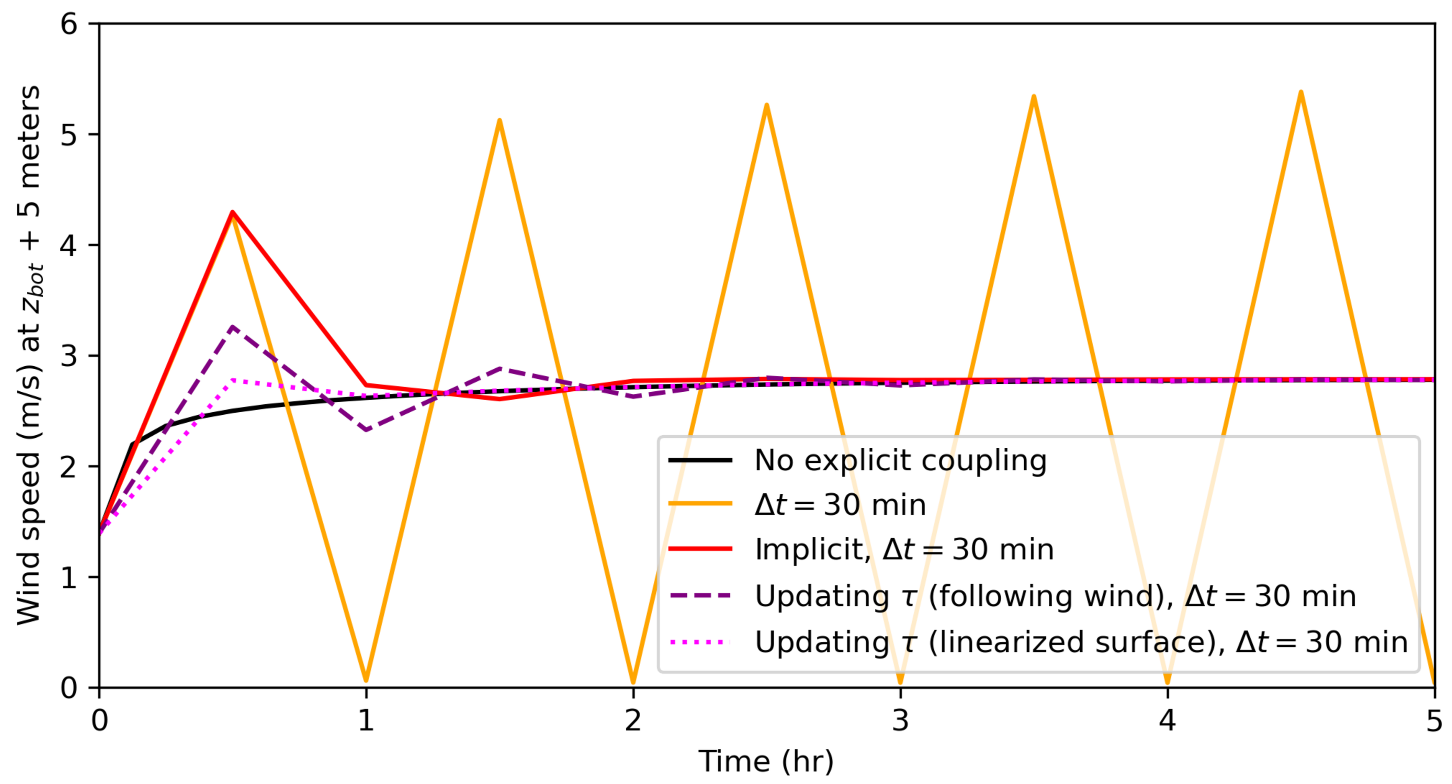


Figure 2: As in Figure 1, but showing three alternative flux coupling methods that prevent numerical instability.

Preliminary E3SMv3 tests

Preliminary tests with E3SMv3 show that switching from the explicit to implicit flux coupling removes the most severe wind oscillations, especially over cold/rough regions (Fig. 3 and 4).

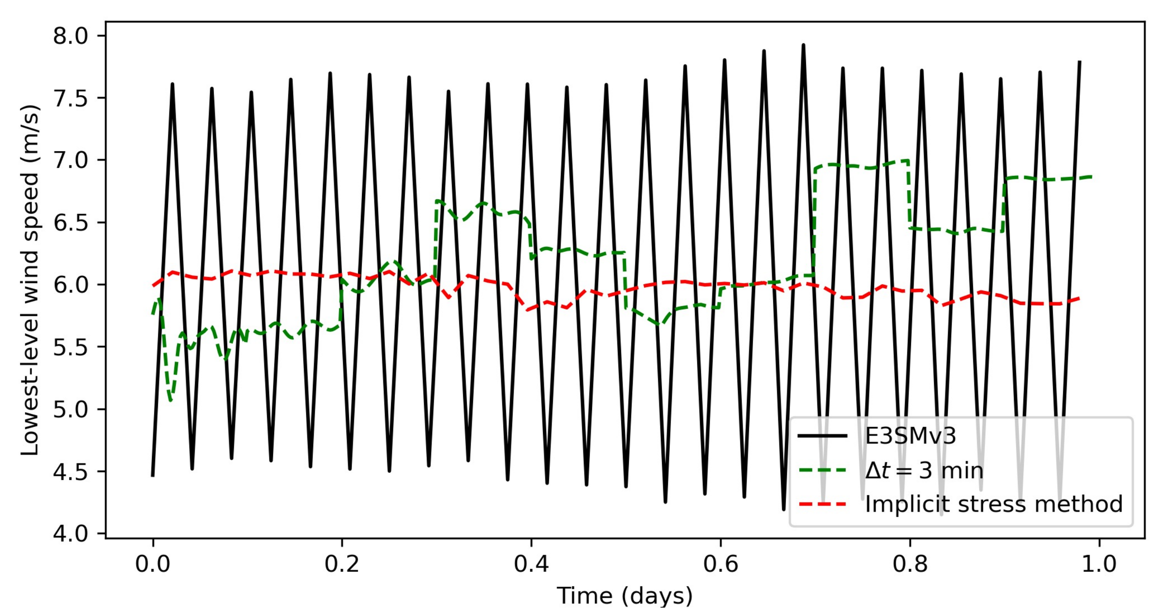


Figure 4: Time series of E3SMv3's near-surface wind for a grid point with oscillations over Greenland, day 4. Compares the default configuration, a shortened (3 min) time step, and implicit coupling.

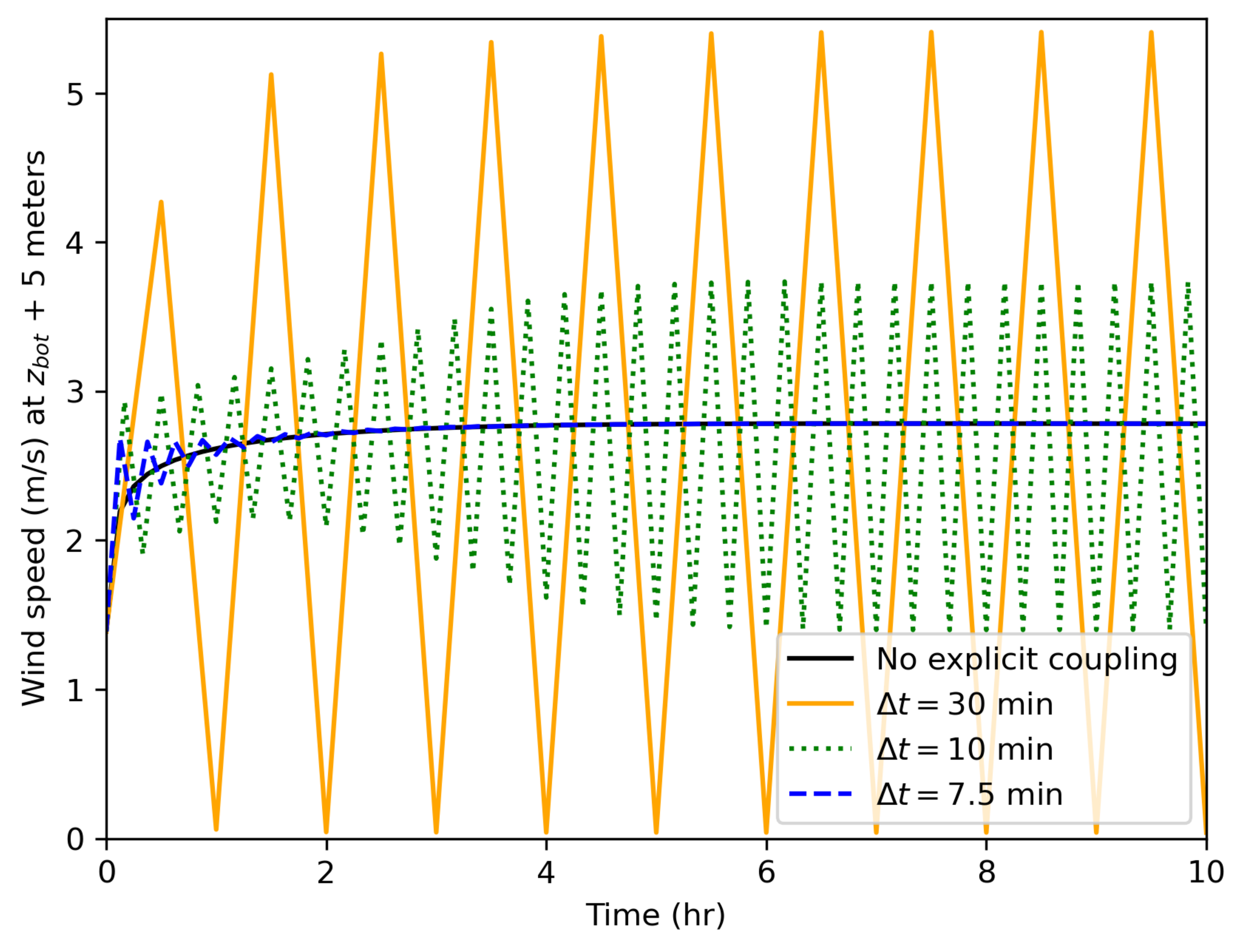


Figure 1: Time series of lowest-model-level wind speed using either an exact calculation (no explicit coupling), or explicit coupling with different time step sizes. The initial condition is exponential in z, with s=0 at the surface.

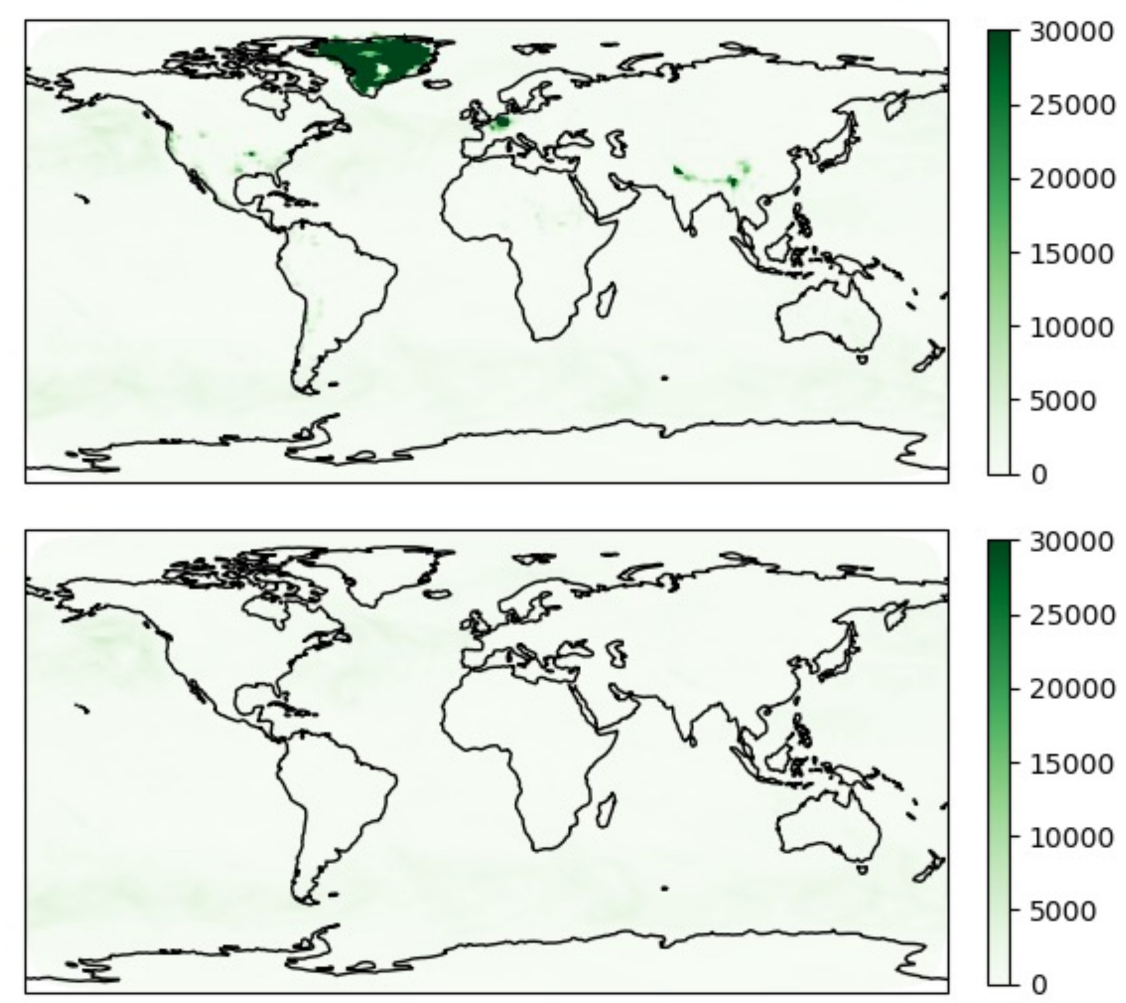


Figure 3: Power spectral density of $2\Delta t$ frequency in near-surface wind for explicit (top) and implicit (bottom) flux coupling, based on 10 day (January) runs of E3SMv3. Maximum value $3\times10^4 m^2/s$ is comparable to the diurnal cycle over most of the globe.

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